PARTICLE CREATION BY A BLACK HOLE AS A CONSEQUENCE OF THE CASIMIR EFFECT

R.M. NUGAYEV and V.I. BASHKOV
Department of Physics, Kazan State University, Kazan 420008, USSR

Received 6 November 1978

It is shown that the effect of particle creation by a black hole is a consequence of the Casimir effect for a spherical shell. The temperature of black-body radiation coincides with that obtained by Hawking.

A. Casimir [1] showed that the vacuum fluctuations of an electromagnetic field give rise to an attractive force between a pair of uncharged conducting parallel plates. The energy responsible for this force is

$$\Delta E = -\pi^2 \hbar c A/720 d^3 , \qquad (1)$$

where A is the area of each plate, and d is the distance between them.

The stress tensor of the vacuum between the plates was calculated in ref. [2]:

$$T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} = \frac{\pi^2 \hbar c^2}{720 d^4} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \frac{3\Lambda^4 \hbar c^2}{\pi^2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix} . \qquad (2)$$

$T_1^{\mu\nu}$ can be interpreted as corresponding to a gas with rather bizarre properties [2]: negative energy density and pressure in the x^3 direction (perpendicular to the plate surfaces), positive pressure in x^1 and x^2 directions. The "gas" satisfies the thermodynamic law $\mathrm{d}E = T\mathrm{d}S - p\mathrm{d}v$, that is why if one slowly ($\mathrm{d}S = 0$) pulls the conductors apart the work done against the tension shows up exactly as an increase in the vacuum energy.

The cutoff dependent part $T_2^{\mu\nu}$ has the same form as the stress tensor of a usual photon gas. It is not responsible for the effect observed in the laboratory [3] and is usually neglected.

Refs. [2] and [4] present a method for investigating the vacuum inside the uncharged sphere made from a physically realizable conductor. A real conductor will conduct well at low frequencies, but its conductivity will diminish as the frequencies increase. The cutoff parameter Λ – the frequency at which the reflection is exchanged by the absorption – should cut off the short waves.

Approximate a sphere of radius d by two parallel plates of area πd^2 at a distance d apart. Using (1) and (2) we obtain:

$$\Delta E = -\pi^3 \hbar c/720 d + 3\hbar c \Lambda^4 d^3/\pi . \qquad (3)$$

The second part is a correction for the finite conductivity of the plates. Detailed calculations [4] for a sphere with ideal conductivity show that ΔE coincides in magnitude with the first part in (3). Only the sign changes. That is why for finite conductivity

$$\Delta E = \pi^3 \hbar c/720 d - 3\hbar c \Lambda^4 d^3/\pi .$$

B. Investigating the behaviour of massless scalar waves in the gravitational field of a nonrotating black hole, Price [5] showed that the curvature of space–time creates an effective potential barrier penetrable for the high-frequency waves and impenetrable for waves with low frequency. The barrier is so well localized near $r = 1.5\, R_g$ ($R_g = 2ZM/c^2$) that for propagation of scalar and electromagnetic waves we can consider the regions quite near the horizon and far away from it as "flat". All the scattering takes place in the small region near $r = 1.5\, R_g$.